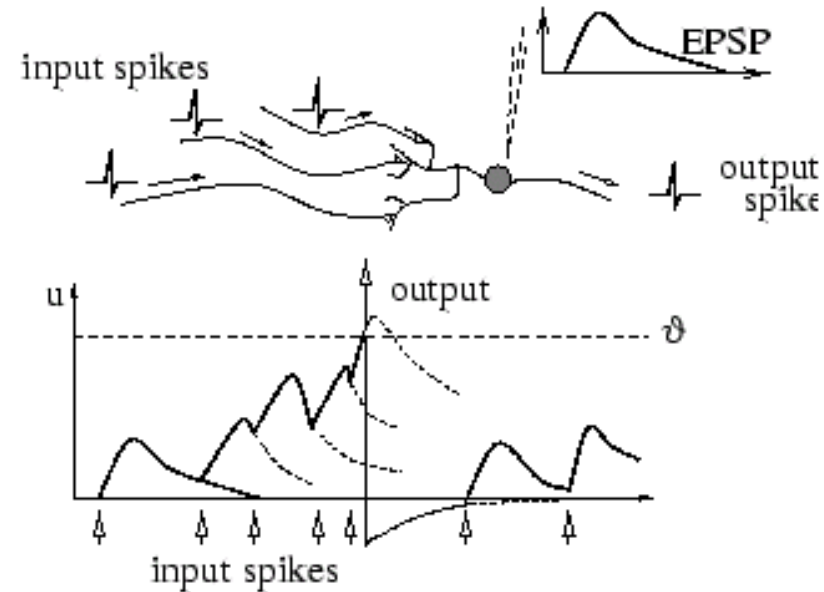


The SRM is a phenomenological model of a spiking neuron



$$\begin{aligned}
 u_i(t) = & u_{rest} + \eta(t - \hat{t}_i) + \sum_j w_{ij} \sum_f \epsilon_{ij}(t - \hat{t}_i, t - t_j^{(f)}) + \\
 & + \int_0^\infty \kappa(t - \hat{t}_i, s) I^{ext}(t - s) ds
 \end{aligned}$$

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The resting potential u_{rest} can be set to 0.

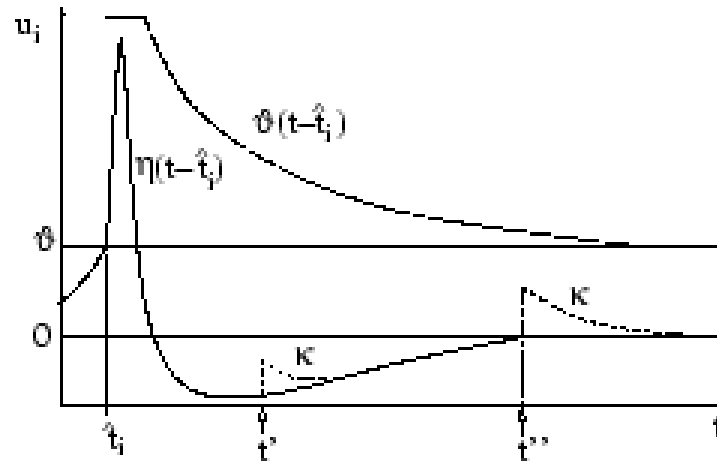
The last spike time of the neuron is written as $\hat{t}_i = \max\{t_i^{(f)} < t\}$.

The **reset kernel** η models the reset after a spike.

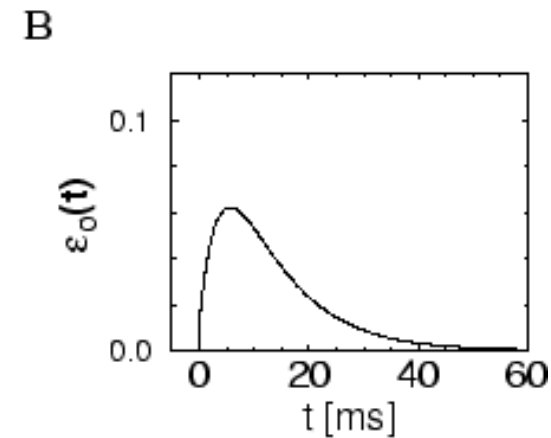
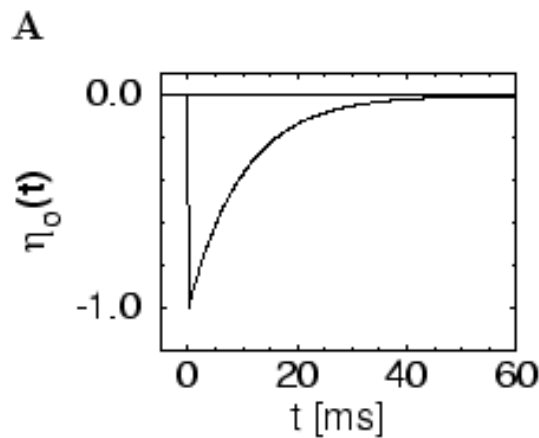
The **response kernel** ϵ models a postsynaptic potential.

κ is the linear response to a current pulse.

The **threshold** ϑ can be defined in dependence on the last spike time. However, this can always be absorbed by the reset kernel, so that we can assume a constant threshold.



Typical shapes for the **reset kernel** and **response kernel**:



Motoneurons

- slowly return to the resting potential after an action potential (time constant ca. 100 msec).
- have a fast response to input currents
- have a reduced response to current pulses after an action potential

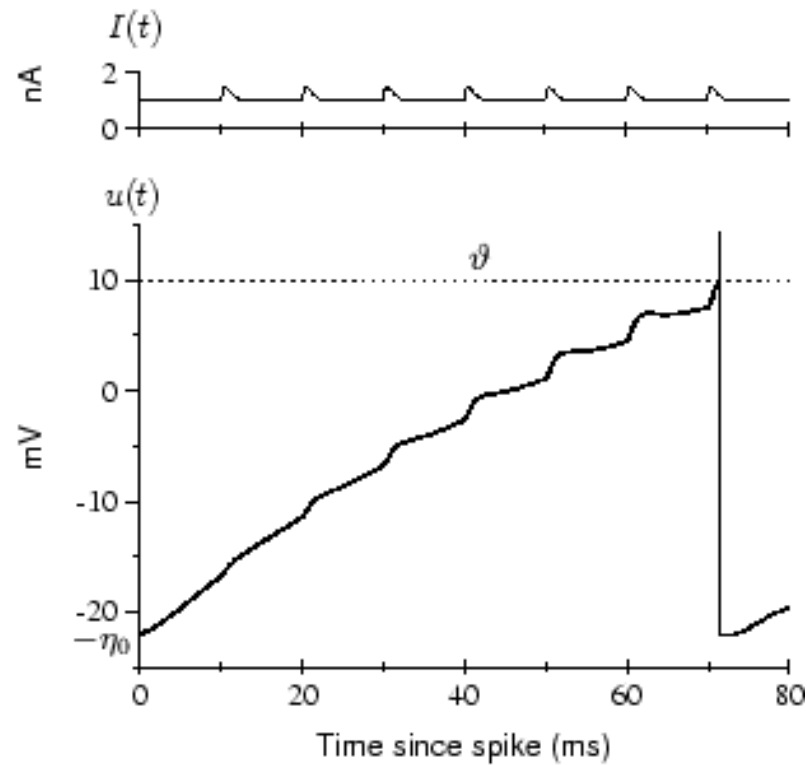
We use the following kernels:

$$\begin{aligned}\eta(t - \hat{t}) &= -\eta_0 e^{-(t-\hat{t})/100ms} \Theta(t - \hat{t}) \\ \kappa(t - \hat{t}, s) &= \frac{R}{4ms} \left[1 - e^{-(t-\hat{t})/100ms} \right] e^{-s/4ms} \Theta(s) \Theta(t - \hat{t} - s).\end{aligned}$$

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[t]

The SRM formalism includes the Integrate-and-Fire model

$$\tau_m \frac{du_i}{dt} = -u_i(t) + R \sum_j w_{ij} \sum_f \alpha(t - t_j^{(f)}) + R I_i^{\text{ext}}(t).$$

After integration we get:

$$\begin{aligned} u(t) &= u_i \exp\left(-\frac{t - \hat{t}_i}{\tau_m}\right) \\ &+ \sum_j w_{ij} \sum_f \frac{1}{C} \int_0^{t - \hat{t}_i} \exp\left(-\frac{s}{\tau_m}\right) \alpha(t - t_j^{(f)} - s) ds \\ &+ \frac{1}{C} \int_0^{t - \hat{t}_i} \exp\left(-\frac{s}{\tau_m}\right) I_i^{\text{ext}}(t - s) ds \\ &= \eta(t - \hat{t}_i) + \sum_j w_{ij} \sum_f \epsilon(t - \hat{t}_i, t - t_j^{(f)}) + \int_0^\infty \kappa(t - \hat{t}_i, s) I_i^{\text{ext}}(t - s) ds, \end{aligned}$$

For a given I&F neuron, we can define the kernels of an equivalent SRM neuron as

$$\eta(s) = u_I \exp\left(-\frac{s}{\tau_m}\right),$$

$$\epsilon(s, t) = \frac{1}{C} \int_0^s \exp\left(-\frac{t'}{\tau_m}\right) \alpha(t-t') dt',$$

$$\kappa(s, t) = \frac{1}{C} \exp\left(-\frac{t}{\tau_m}\right) \Theta(s-t) \Theta(t).$$

In this simplified model, we neglect the dependence of κ and ϵ on the last action potential:

$$u_i(t) = \eta(t - \hat{t}_i) + \sum_j w_{ij} \sum_f \epsilon_0(t - t_j^{(f)}) + \int_0^\infty \kappa_0(s) I^{ext}(t - s) ds.$$

With the SRM_0 model, I&F neurons cannot be modeled exactly.

Relatively simple models:

Integrate-and-Fire: Very well suited for simulations. One can simulate large networks efficiently.

SRM: Suited for mathematical analysis.

The models are often too complex for detailed mathematical analysis. In particular the non-linearity due to the threshold and the reset causes problems. Here, one often uses simpler models like the linear Poisson neuron (spiking, but linear and stochastic).

The dynamics only depends on the *last* action potential of the neuron: One is unable to model complex firing behavior, e.g.,

Frequency adaptation: Many neurons adapt their output rate to the rate of the inputs. First, they respond strongly to increased input, then the output frequency adapts to a low rate.

Bursting: Many neurons are *bursting*. Bursts are short series of action potential of high frequency. On a constant input current, such neurons respond with burst - silence - burst -

Stuttering: Such neurons respond to a constant input current with irregular spike trains.

Channel kinetics: The exact behavior and interactions of channels is not modeled. Note that the opening of many channels leads to a smaller R_m and therefore to a smaller τ_m . → Hodgkin-Huxley models.

Spatial structure: The magnitude of PSPs depends on the *local* voltage at the synapse. The voltage distribution on a real neuron is complex. → Compartmental models.