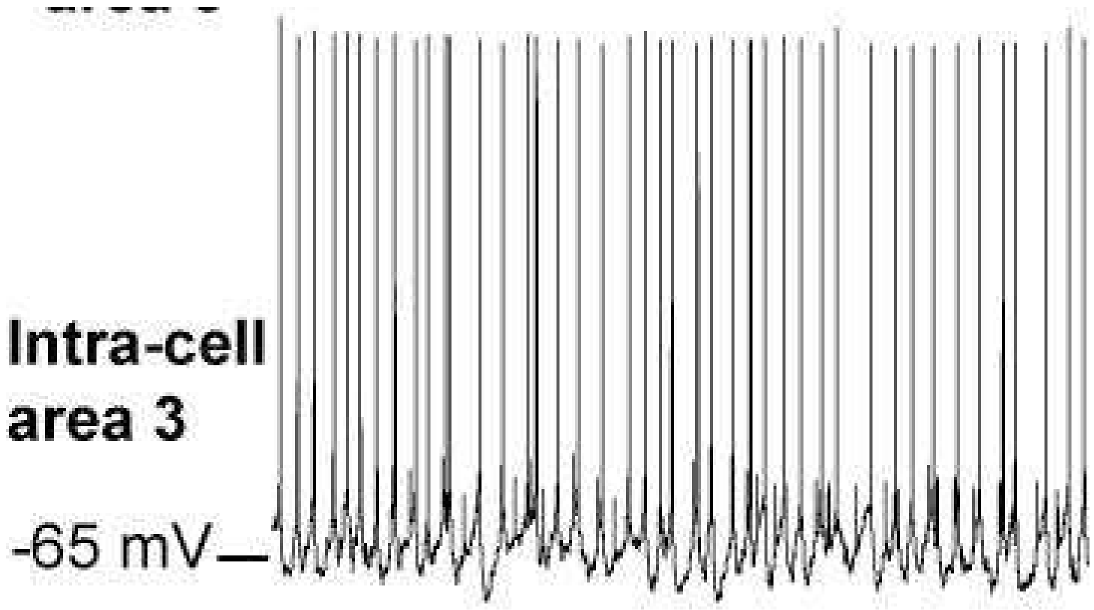
Recording

Intracellular recording of a somatosensory neurons in the cat.



Features:

- The ***membrane potential*** $u(t)$ is modeled
- The ***spike generation mechanism*** is implemented by a threshold
- Inputs und Outputs are spike trains

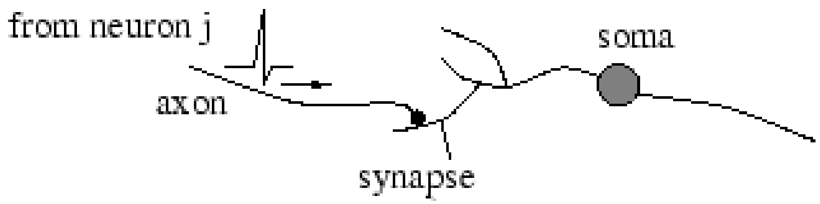
The spike train of a neuron i can be formalized as a list of spike times $\langle t_i^{(1)}, t_i^{(2)}, \ldots, t_i^{(f)}, \ldots \rangle$,
or as a sum of Dirac delta pulses

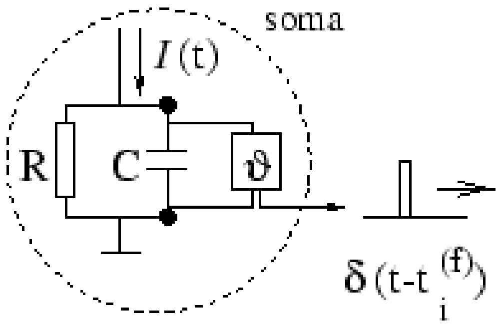
$$S_i(t) = \sum_f \delta(t - t_i^{(f)}).$$

The Integrate-and-Fire Model

Threshold model: When the membrane potential $u(t)$ crosses the threshold ϑ from below, a spike is triggered.

The membrane potential $u(t)$ is given by a simplified model of the cell membrane.





Membrane potential

$R_m \ldots$ membrane resistence

$C_m \ldots$ membrane capacity

$I(t) \ldots$ input current

$$I(t) = I_{ext}(t) + I_{syn}(t) + I_{noise}.$$

The membrane potential is then given by (the resting potential is assumed at 0)

$$\tau_m \frac{du(t)}{dt} = -u(t) + R_m I(t),$$

where $\tau_m = R_m C_m$ is the membrane time constant.

Spiking mechanism

When the membrane voltage $u(t)$ exceeds the threshold ϑ, then

- the voltage is reset to the ***reset voltage*** $u_r < \vartheta$,
- and stays there for the ***absolute refractory period*** Δ_{abs}.

Solution for time varying current:

Let $\hat{t}$ denote the last spike time of the neuron before t.

When the differential equation is solved with $u(\hat{t}) = u_r$, one gets for $t > \hat{t} + \Delta_{abs}$

$$u(t) = u_r e^{-\frac{t-\hat{t}-\Delta_{abs}}{\tau_m}} + \frac{1}{C_m} \int_0^{t-\hat{t}-\Delta_{abs}} e^{-\frac{s}{\tau_m}} I(t-s) ds$$

- $I(t)$ is filtered with the membrane time constant.
- Note that the integration restarts with every post-spike.

Each presynaptic spike generates a current input of some given form $\alpha(t)$.

$$I_{syn}(t) = \sum_j w_{ij} \sum_f \alpha(t - t_j^{(f)}) = \sum_j w_{ij} \int_{-\infty}^{t} \alpha(t-s) S_j(s) ds.$$

Commonly used forms for α:

- exponential: $\alpha(s) = A \exp(-s/\tau_s)\Theta(s)$.
- “Alpha”-function: $\alpha(s) = A[\exp(-s/\tau_s) - \exp(-s/\tau_r)]\Theta(s)$, where τ_r is the time constant for the rise, and τ_s is the time constant for the decay of the current.

Synaptic Parameters:

excitatory, AMPA: exponential with $\tau_s = 5$ms.

excitatory, NMDA: ”alpha” with $\tau_r = 2$ms and $\tau_s = 150$ms.

inhibitory: ”alpha” with $\tau_r = 0.5$ms and $\tau_s = 10$ms.

Conductance-based Synapses

- An active synapse changes the conductance of the membrane for some ion by opening postsynaptic channels.
- Therefore, the current through a synaptic channel depends on the membrane voltage at the channel.
- For each type of synapse, one can define the *reversal potential* E_{syn} where no current flows even if the synapse is active.
- An active synapse thus changes the conductance $g(t - t^{(f)})$.

The current for a spike at time $t^{(f)}$ is given by

$$\alpha(t - t_j^{(f)}) = -g(t - t_j^{(f)})[u_i(t) - E_{syn}].$$

For excitatory synapses, E_{syn} is much higher than the threshold, for inhibitory synapses, E_{syn} is close to the resting potential.